nature photonics

SUPPLEMENTARY INFORMATION

https://doi.org/10.1038/s41566-018-0266-5

In the format provided by the authors and unedited.

# Nanophotonic optical gyroscope with reciprocal sensitivity enhancement

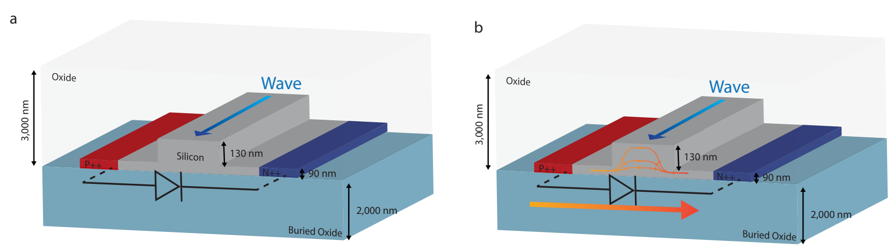
**Parham P. Khial *, Alexander D. White and Ali Hajimiri**

Department of Electrical Engineering, California Institute of Technology, Pasadena, CA, USA. *e-mail: pporsand@caltech.edu

# Supplementary Information

# PIN Diode

PIN diodes are active elements that can be controlled by external electronic circuitry in order to apply an arbitrary phase shift (i.e., time delay) to incident light. The diode is constructed from a silicon waveguide slab squeezed between $P^{++}$ and $N^{++}$ etched doping (Supplementary Figure 1), thus forming a $P^{++}$–Intrinsic silicon–$N^{++}$ (PIN) structure. When the diode is forward-biased, carriers (including both electrons and holes) pass perpendicularly (relative to the direction of the propagating wave) through the waveguide. These carriers change the effective index of intrinsic silicon , thereby decreasing the speed of light. For a fixed waveguide length, a higher injected current introduces more phase shift. The measured turn-on voltage of the diode is 0.8 V, and 30 mA is needed for $2\pi$ phase shift.




Supplementary Figure 1: a. PIN diode in the "off" state. b. PIN diode in the "on" state, showing current injected into the silicon waveguide.

# Ring Resonator Design and Measurement

Although ring resonators provide gain, they come with the drawback of needing to be tuned. Process variations and other imperfections shift as well as distort the ring's response. We designed two ring resonators, both with a radius of 500 $\mu$m (Supplementary Figure 2a). In order to reduce loss and improve the resonator response, we made the waveguide width 1.2$\mu$m (instead of 500nm).By using 1.2 $\mu$m waveguide the effects of loss and reflection are minimized and the response of the rings are measured (Supplementary Figure 2a). Next, using the specified loss of the waveguide (0.35 dB/cm), we designed the coupling regions. Due to fabrication mismatch, additional resistors were placed inside the rings to enable tuning of the resonators [1, 2]. Tuning is achieved by increasing the local temperature around the resistor, which increases the effective index of silicon and decreases the wave propagation speed in that region, thereby introducing a phase shift (Supplementary Figure 2b). For minimization of shot noise, the resistors are designed to be as large as possible.

## Tuning Process

Each of the two rings contains a tuning resistor. The outputs of the rings are added together. By sweeping the resistances of the two resistors, we simultaneously tune both ring resonators by maximizing their combined output when $\varphi_{\text{applied}} = 0$ (Supplementary Figure 3a).

## Calibration Requirement For Reciprocal Sensitivity Enhancement

Despite the fact that all ring mismatches are accounted for in equation (3), there are other mismatches in the system that are not captured. For example, a phase detector is needed for each direction (Figure 2b), and these detectors could have different characteristics. Let $\gamma$ be the overall relative amplitude mismatch between the two directions. To negate the effect that $\gamma$ has, we use a weighted combination of the two signals:

$$\mathrm{Y} = \begin{cases} \mathrm{A}_1\gamma \left[+\mathrm{X}(t) \left|\Delta\varphi_{\text{Sagnac}}\right| + \mathrm{U}(t)\right], & 0 < t < \dfrac{T}{2} \\ \mathrm{A}_2 \left[-\mathrm{X}(t) \left|\Delta\varphi_{\text{Sagnac}}\right| + \mathrm{U}(t)\right], & \dfrac{T}{2} < t < T, \end{cases} \tag{1}$$

where $\mathrm{A}_1$ and $\mathrm{A}_2$ are the applied gains; to compensate for mismatch, we need

$$\mathrm{A}_1\gamma = \mathrm{A}_2. \tag{2}$$

Due to the finite precision of the VGAs, satisfying equation (8) ($\mathrm{A}_1\gamma = \mathrm{A}_2$) exactly, which would theoretically cancel all of the noise due to amplitude mismatch, is not practically possible. That being said, we can figure out how precise $\mathrm{A}_1$ and $\mathrm{A}_2$ need to be for a minimum detectable rotation rate. This value can be calculated and is plotted in Supplementary Figure 3b.

## Allan Deviation Plot for Gyroscopes' Performance

Allan variance analysis originally was developed to characterize the stability of the oscillators. However, it can be used to determine the intrinsic noise in time-varying systems as well [3]. Generally speaking, Allan deviation plot consists of Allan variance value for different averaging time and it usually is reported by two values: Angle Random Walk (ARW) and Bias Instability (BIS) with units of $^\circ/\sqrt{\mathrm{hr}}$ and $^\circ/\mathrm{hr}$ respectively.

Allan variance can be calculated in terms of output rate (which usually is proportional to the actual output in rate gyros) or output angle. These two quantities are related as below:

$$\theta(t) = \int_0^t \Omega(t)dt \tag{3}$$

And Allan variance can be calculated by using this formula:

$$\begin{aligned} \sigma^2(\tau) &= \frac{1}{2} < (\overline{\Omega}_{k+m} - \overline{\Omega}_k)^2 > \\ &= \frac{1}{2\tau^2} < (\theta_{k+2m} - 2\theta_{k+m} + \theta_k)^2 > \end{aligned} \tag{4}$$

Where $\tau = m\tau_0$ and $\tau_0$ is the sampling period.

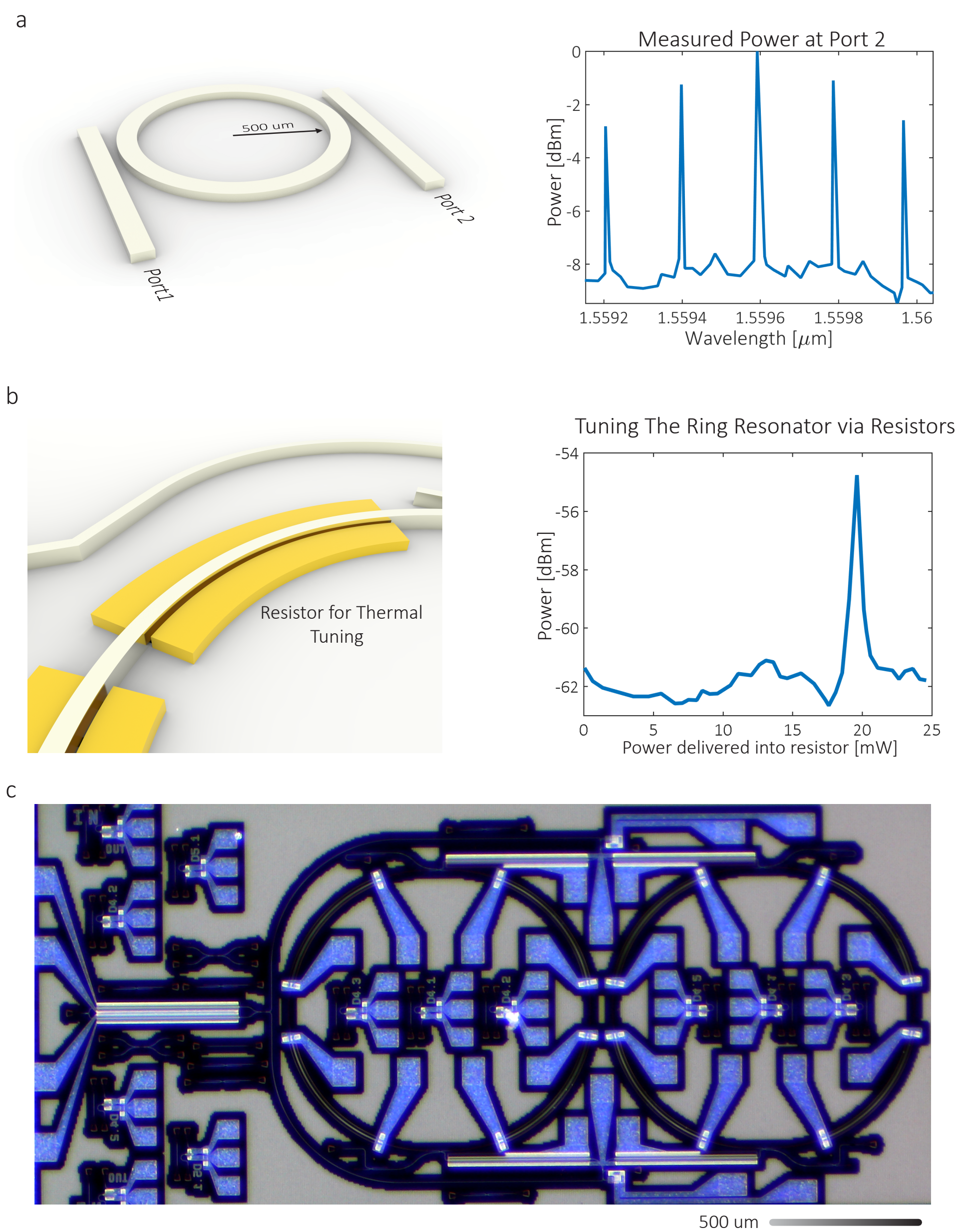
In a typical Allan deviation plot (Supplementary Figure 4) ARW fit and BIS fit are shown.

# References

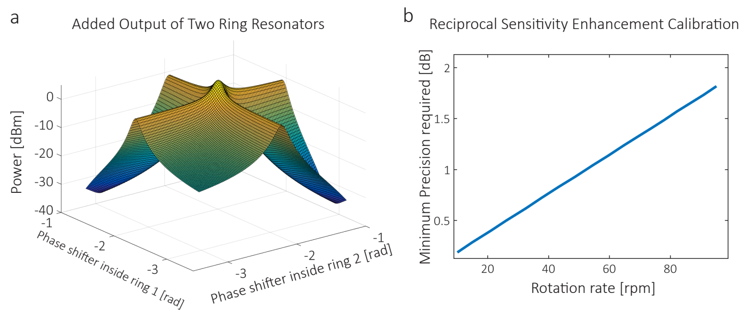

[1] Zortman, W. A., Trotter, D. C., & Watts, M. R. Silicon photonics manufacturing. *Optics express*, 18(23), 23598-23607 (2010).

[2] Chrostowski, L., Wang, X., Flueckiger, J., Wu, Y., Wang, Y., & Fard, S. T. Impact of fabrication non-uniformity on chip-scale silicon photonic integrated circuits. In Optical Fiber Communication Conference. *Optical Society of America.*, (2014).

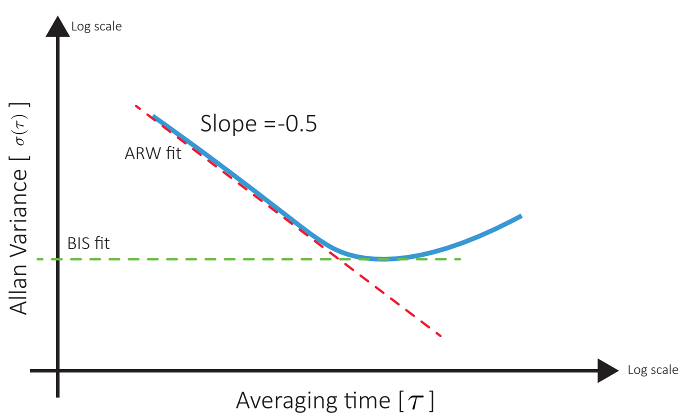
[3] IEEE Standard Specification Format Guide and Test Procedure for Single-Axis Laser Gyros *IEEE Std* 647-2006 (Revision of IEEE Std 647-1995) , vol., no., pp.0 1-83, (2006).




Supplementary Figure 2: a. Designed micro-ring resonator and measured response, which shows an FSR of 25 GHz and $Q = 100{,}000$. The input light was fed into port 1 and the output was taken from port 2. b. Additional resistor added inside the rings and measured tuning response, depicting the coupled power versus the power burned by the resistors. c. Die Photo of the fabricated device.




Supplementary Figure 3: a. Output power of the two rings versus the resistances in rings 1 and 2. b. Estimated minimum required precision versus minimum detectable rotation rate for 5% mismatch in fabrication.




Supplementary Figure 4: A typical Allan deviation plot with ARW fit and BIS fit. ARW is equal to the value of the plot at $\tau = 1$.